Optimal economic behaviour in a fractal city versus a non fractal (thünian) city

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Outline

1. Introduction
2. Economic model
3. Geographical model
4. Simulation
5. Conclusions

Introduction (1)

- Fundamental question: economic landscape formation in urban sprawl
- Land use: mix of industries, households, agriculture, green areas
- Actors: households in search of a dwelling place
- Decision influenced by lot size, land price, job accessibility, urban and rural amenities
- Amenities are public goods available at given places
- Dissatisfaction with classical (thünian, Muth-Alonso-like) urban economics

Introduction (2)

- Heterogenous space: mix of urban and green areas
- Hierarchy of urban and rural amenities
- Extension of Cavailhès et al. (EPA 2004): bring together geography and economics
- Household location problem derived from urban economics
- Geographical space: multifractal Sierpinski carpet
- Simulate the impact of variations in the economic parameters or in the behavior of the households

Economic model (1)

Household's annual income W is allocated to:

- renting a lot of size Z for a price R
- N journeys-to-work to the CBD located at a distance d_K
- purchasing a quantity X of a composite good (taken as the numeraire)
- moving to sites where urban and rural amenities can be enjoyed
- t is the unit transportation cost, whatever the type of travel

Economic model (2)

Urban and rural amenities:

- K levels of urban amenities: $k = 1, \ldots, K$
- $k = 1$ means proximity services, $k = K$ means highest level, available only at the CBD
- provided free of charge at respective distances $d_1, \ldots, d_K$
- respective numbers of annual trips: $a_1, \ldots, a_K$
- K levels of rural amenities: $k = 1, \ldots, K$
- provided free of charge at respective distances $e_1, \ldots, e_K$
- respective numbers of annual trips: $b_1, \ldots, b_K$

Economic model (3)

Household's budget constraint:

$$W = X + RZ + tNd_K + t\sum_{k=1}^{K} d_k a_k + t\sum_{k=1}^{K} e_k b_k \quad (1)$$

Cobb-Douglas utility function:

$$U = \frac{1}{\alpha^\alpha \beta^\beta \gamma^\gamma \delta^\delta} X^\alpha Z^\beta A^\gamma B^\delta \quad (2)$$

- X: composite good
- Z: plot size
- A: urban amenities, differentiated product.
- B: rural amenities, differentiated product.
- $\alpha + \beta + \gamma + \delta = 1$

Economic model (4)

CES (constant elasticity of substitution) for the urban amenities:

$$A = \left(\sum_{k=1}^{K} a_k^{\rho} \right)^{\frac{1}{\rho}} \tag{3}$$

- a_k: annual number of trips to enjoy level k amenity
- ρ: taste for variety for urban amenities, $-\infty < \rho \leqslant 1$

Idem for the rural amenities:

$$B = \left(\sum_{k=1}^{K} b_k^{\sigma} \right)^{\frac{1}{\sigma}} \tag{4}$$

- b_k: annual number of trips to enjoy level k amenity
- σ: taste for variety for urban amenities, $-\infty < \sigma \leqslant 1$

Economic model (5)

Two stage resolution method. First solve the quest for urban amenity subproblem:

$$\min_{a_1,\ldots,a_K} \sum_{k=1}^{K} d_k a_k \quad \text{subject to} \quad \left(\sum_{k=1}^{K} a_k^{\rho} \right)^{\frac{1}{\rho}} = A \tag{5}$$

Define the global index of accessibility to urban amenitites:

$$D = \left(\sum_{k=1}^{K} d_k^{\mu} \right)^{\frac{1}{\mu}}, \quad \mu = \frac{\rho}{\rho - 1} \tag{6}$$

Solution

$$a_k = \left(\frac{d_k}{D} \right)^{\frac{1}{\rho-1}} A,\ k = 1, \ldots, K \quad \text{and} \quad \sum_{k=1}^{K} d_k a_k = DA \tag{7}$$

Economic model (6)

Solve the similar subproblem for the rural amenities:

$$\min_{b_1,\dots,b_K} \sum_{k=1}^{K} e_k b_k \quad \text{subject to} \quad \left(\sum_{k=1}^{K} b_k^{\sigma}\right)^{\frac{1}{\sigma}} = B \tag{8}$$

Define the global index of accessibility to rural amenitites:

$$E = \left(\sum_{k=1}^{K} e_k^{\nu}\right)^{\frac{1}{\nu}}, \quad \nu = \frac{\sigma}{\sigma - 1} \tag{9}$$

Solution

$$b_k = \left(\frac{e_k}{E}\right)^{\frac{1}{\sigma-1}} B,\ k = 1, \dots, K \quad \text{and} \quad \sum_{k=1}^{K} e_k b_k = EB \tag{10}$$

Economic model (7)

Second stage: solve the master problem:

$$\max_{X,Z,A,B} \frac{1}{\alpha^\alpha \beta^\beta \gamma^\gamma \delta^\delta} X^\alpha Z^\beta A^\gamma B^\delta \quad \text{s.t.} \quad X + RZ + tDA + tEB = W_D \tag{11}$$

with the disposable income

$$W_D = W - tNd_K \tag{12}$$

Solution

$$X^\star = \alpha W_D, \; Z^\star = \frac{\beta W_D}{R}, \; A^\star = \frac{\gamma W_D}{tD}, \; B^\star = \frac{\delta W_D}{tE} \tag{13}$$

Indirect utility function:

$$U^\star = \frac{W_D}{R^\beta t^{\gamma+\delta} D^\gamma E^\delta} \tag{14}$$

Economic model (8)

- Urban equilibrium in an open city where $\bar{U}$ is the utility of the rest of the world.
- Equilibrium rent:

$$\Psi = \max\left[\left(\frac{W_D}{\bar{U}t^{\gamma+\delta}D^{\gamma}E^{\delta}}\right)^{\frac{1}{\beta}}, R_A\right] \tag{15}$$

where R_A is the (agricultural) opportunity rent.

- When $\Psi > R_A$, the equilibrium lot size is

$$Z = \frac{\beta W_D}{\Psi} = \beta W_D^{1-\frac{1}{\beta}}\left(\bar{U}t^{\gamma+\delta}D^{\gamma}E^{\delta}\right)^{\frac{1}{\beta}} \tag{16}$$

- Parameters of the problem: $(W, t, R_A, \bar{U}, N, \alpha, \beta, \gamma, \delta, \rho, \sigma)$
- Compute $a_k^\star$, $b_k^\star$, $A^\star$, $B^\star$, Ψ, $X^\star$, $Z^\star$, population.

Geographical model (1)

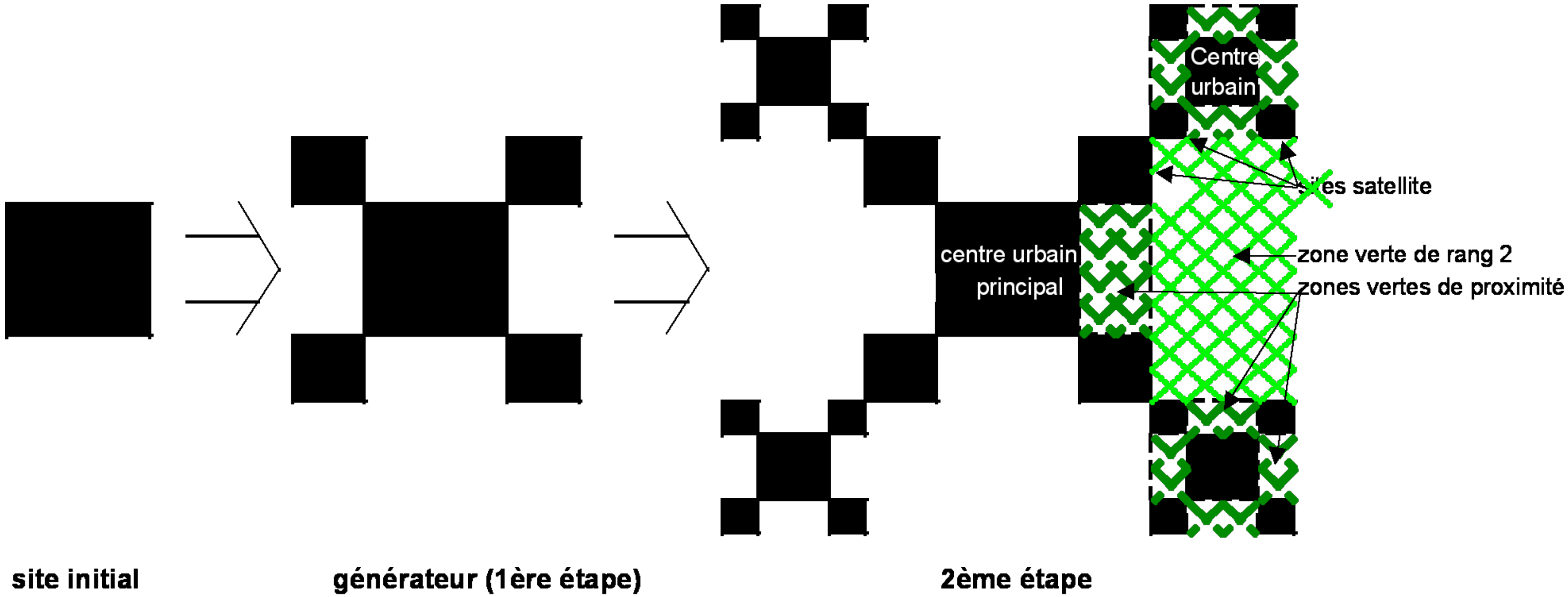

Figure 1: Multifractal Sierpinski carpet: first two iterations

Geographical model (2)

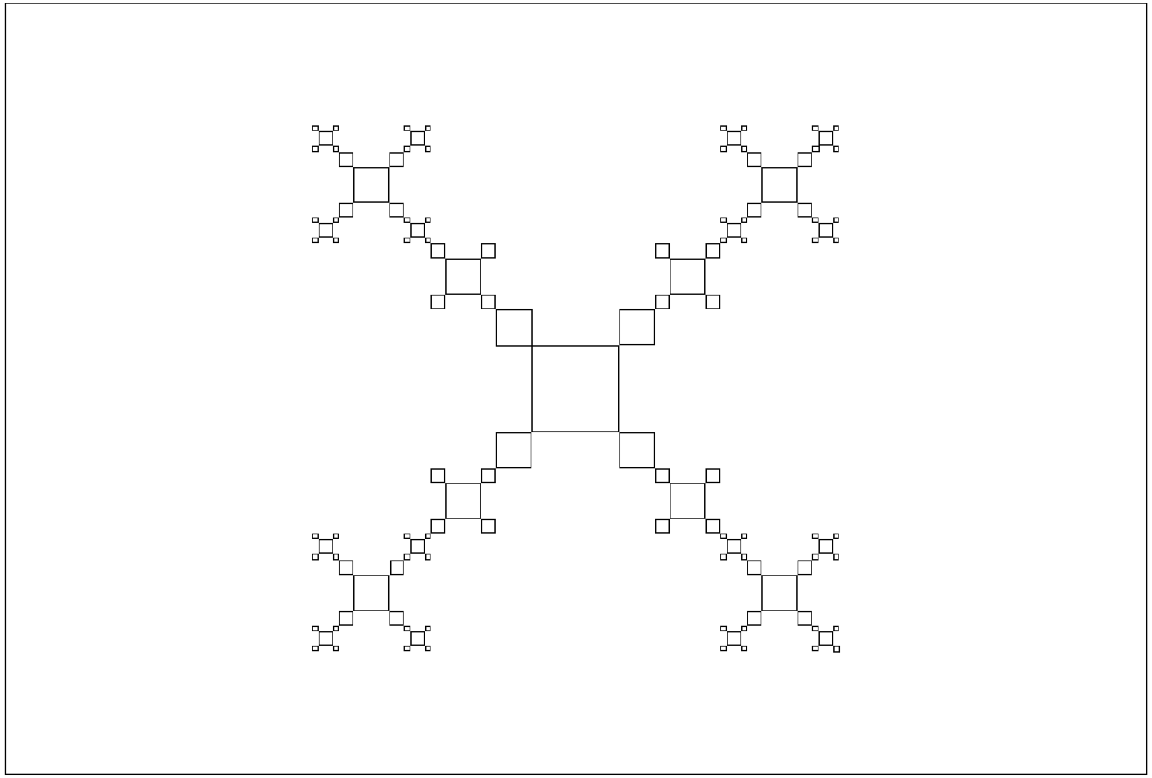

Figure 2: Multifractal Sierpinski carpet: third iteration

Geographical model (3)

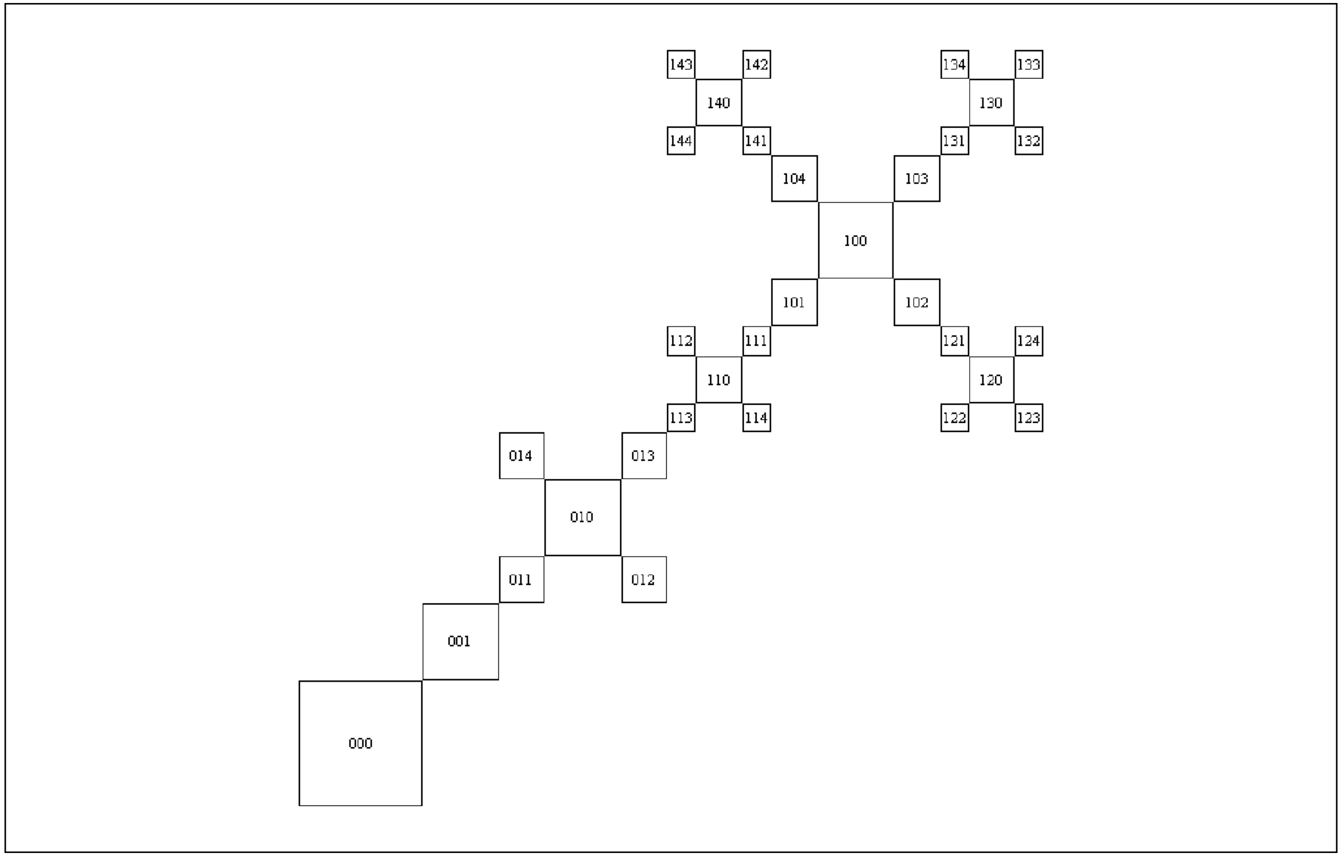

Figure 3: Coding the residential areas (N-E quarter)

Geographical model (4)

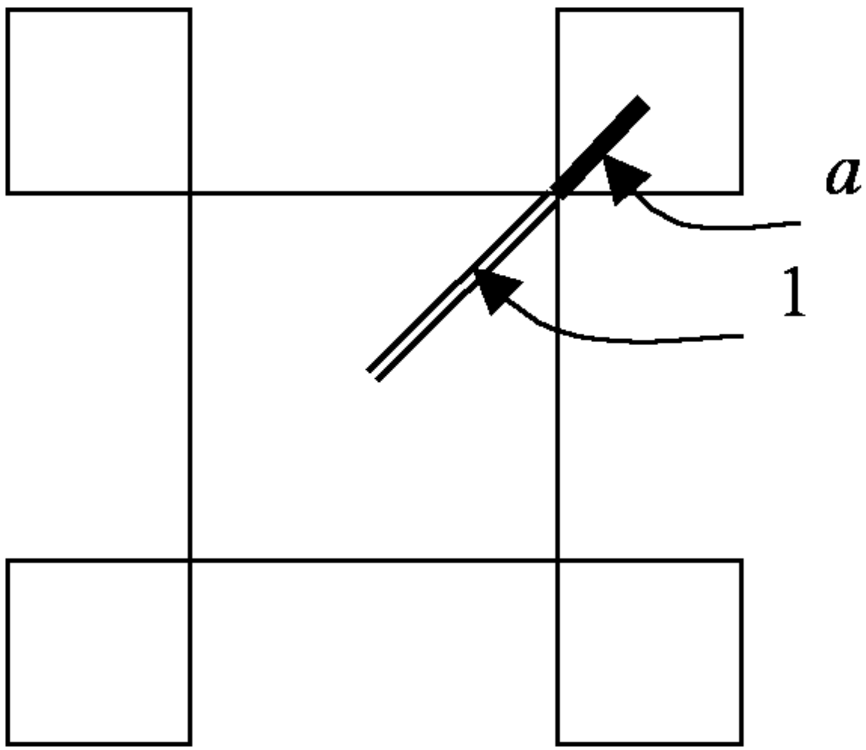

Figure 4: Measure of distances

Geographical model (5)

- $d_k(C_3C_2C_1)$: distance between site $(C_3C_2C_1)$ and urban amenity of rank k.
- $d_k(C_3C_2C_1) = \zeta_k a^{\ell(k)}(1+a)^{m(k)}(1+2a)^{n(k)}$
- $\zeta_3 = 1$
- $\zeta_2 = (-1)^{\delta_{C_2,1}+\delta_{C_3,0}}$
- $\zeta_1 = (1-\delta_{C_1,0})\,(-1)^{\delta_{C_1,1}+\delta_{C_2,1}\left(1-\delta_{C_3,0}\right)\left(\delta_{C_1,1}+\delta_{C_1,C_3}\right)}$
- $\delta_{m,n} = 1$ if $m = n,\ 0$ otherwise
- $\ell(k)$, $m(k)$, $n(k)$ depend on $(C_3C_2C_1)$
- More cumbersome: $e_k(C_3C_2C_1)$

Simulation(1)

- Goal: compare model's prediction with real world observations; make (qualitative) predictions.
- Starting point: 23 French metropolitan areas with population between 100,000 and 200,000 inhabitants.
- Derive $\Psi^o(x)$, $Z^o(x)$, limit of the urban area f^o, population P^o.
- Some parameters of the problem can be observed: W, t, R_A, N, β.
- Arbitrarily set: γ, δ, ρ, σ.
- The rest can be deduced: α, $\bar{U}$.

Simulation (2)

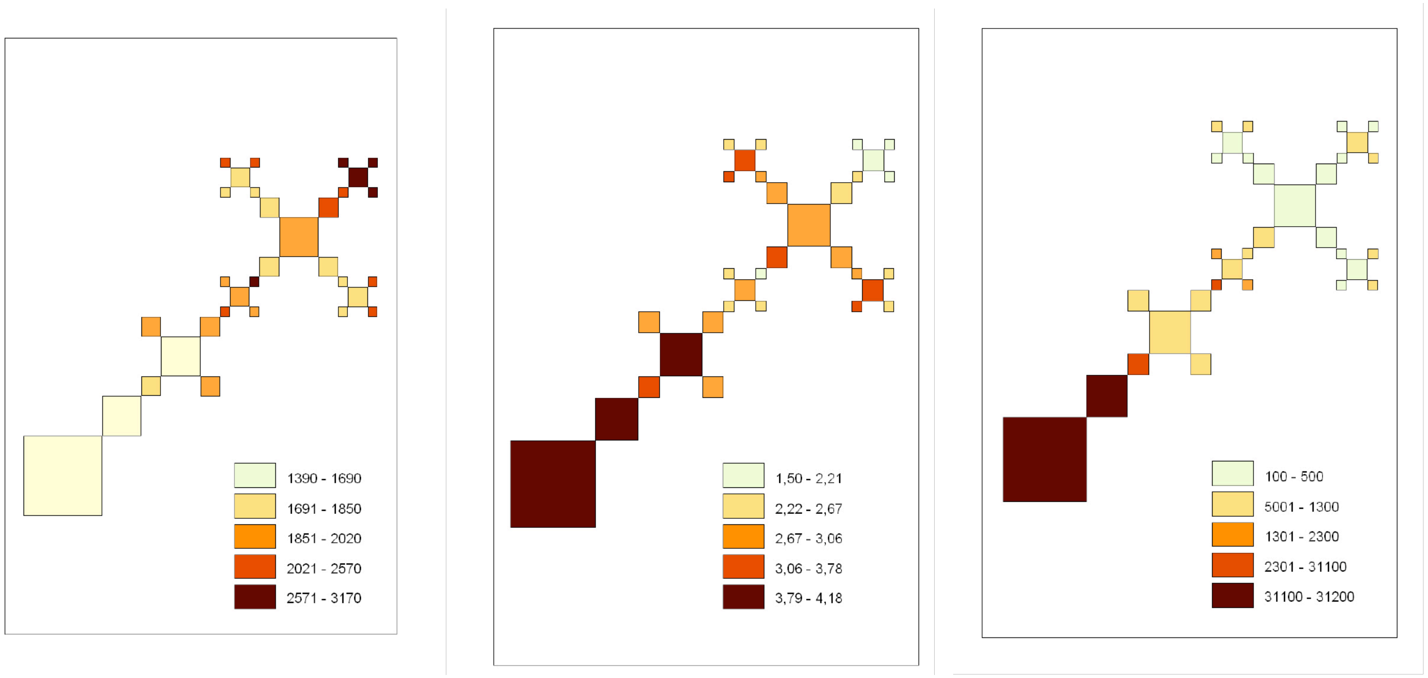

Figure 5: Lot size, equilibrium rent, population distribution on benchmark problem

Simulation (3)

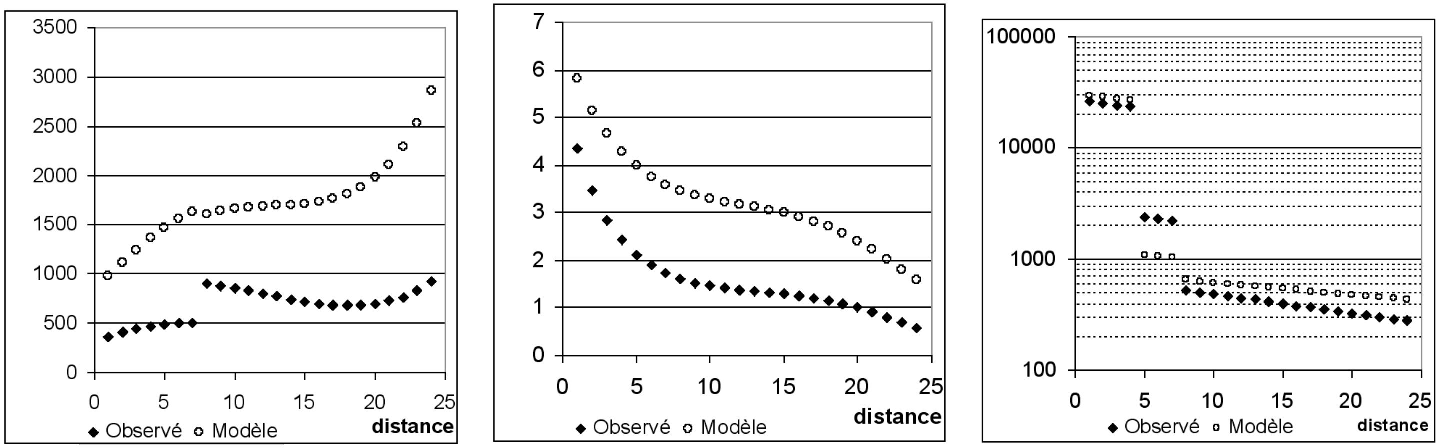

Figure 6: Lot size, rent, population as functions of distance to the CBD. Adjustment by polygon of degree 3. Comparison between model and real world.

Simulation (4)

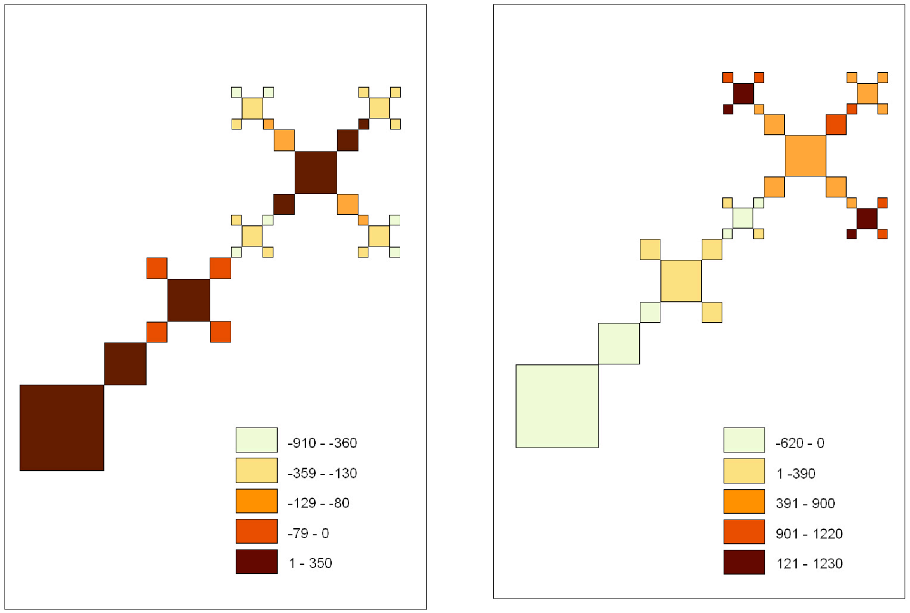

Figure 7: Lot size: comparison between benchmark and more preference respectively for rural and urban amenties

Simulation (5)

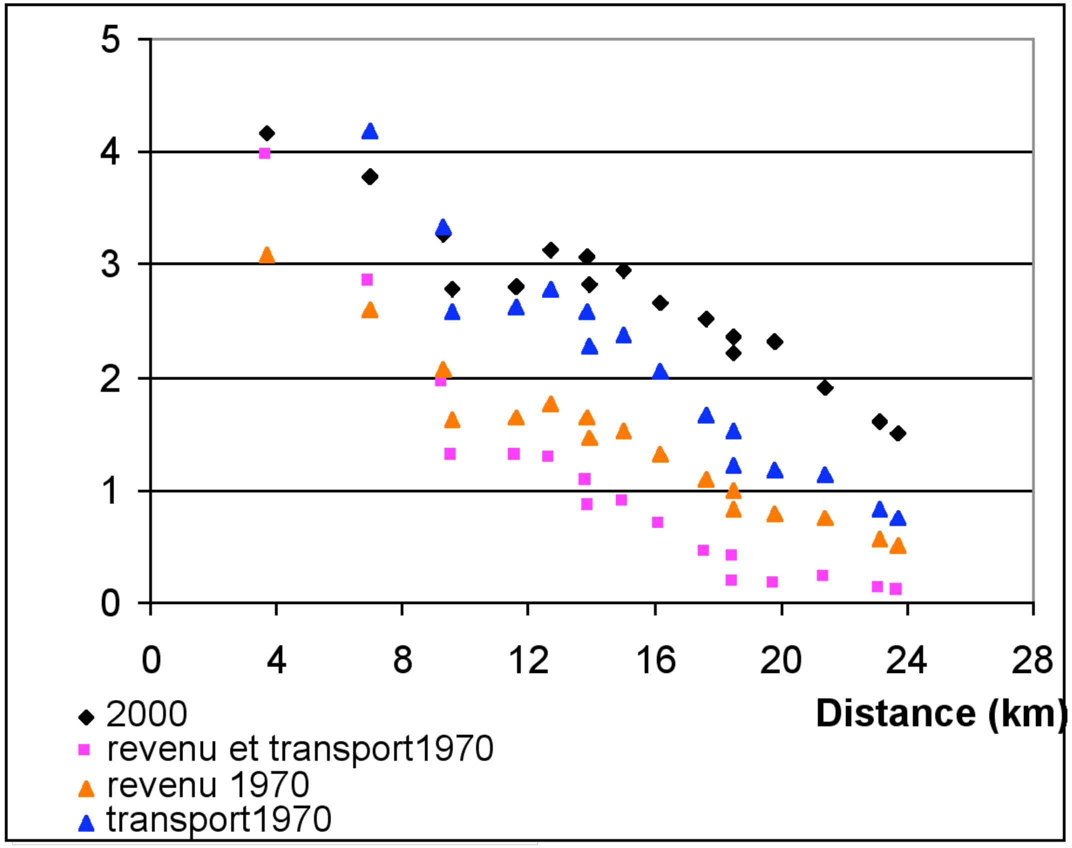

Figure 8: Land rent: evolution of income and/or transportation cost

Simulation (6)

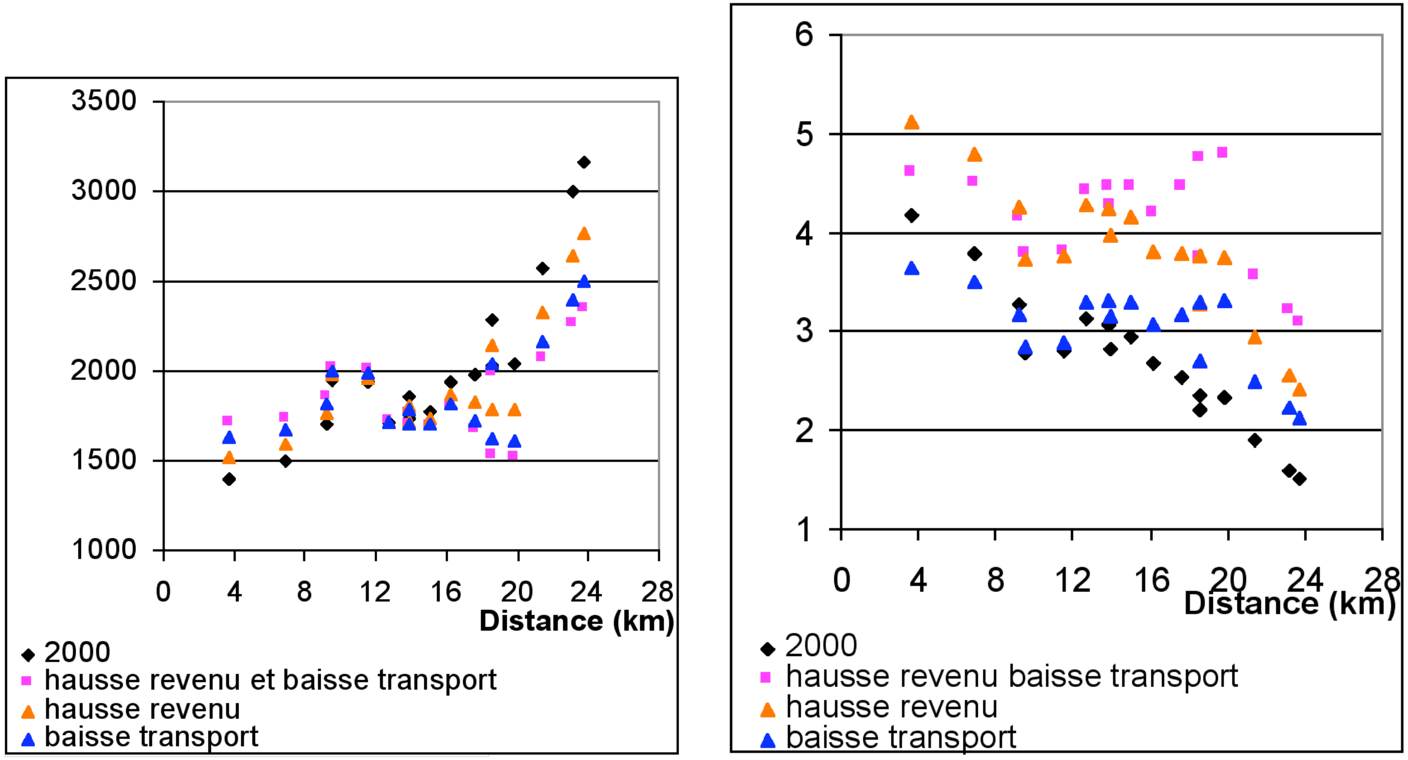

Figure 9: Further evolution of income and/or transportation cost: lot size and land rent

Conclusions

- Breakthrough with classical (thünian, Muth-Alonso-like) urban economics.
- Economics: difficulties to incoporate geographical heterogeneity.
- Geography: models with weak economic background.
- Attempt to bridge the gap between both disciplines.
- Good news: analytically solvable model.
- Allows general qualitative predictions on the evolution of the urban structures depending on changes in the parameters.
- Bad (?) news: remains at a too high level abstraction to deal with real-world applications. Hence necessity to explore either more refined geometric structures and/or frameworks.